ORIE 6180: Design of Online Marketplaces **Spring 2016**

Lecture 11 — March 9

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11.1 Overview of the last lecture

Recall setting from last lecture of a 2-sided platform with symmetric sides L and R.

Each user i in side L arrives with 2 parameters $(b_i^L, B_i^L) \backsim F^L(b_i, B_i)$ where b is their **interaction benefit**, B is their **membership benefit**, and F^L is a distribution with density $f^L(b_i, B_i)$.

User i gains utility u_i^L where:

$$u_i^L = b_i^L N^R + B_i^L - P^L(N^R)$$

and will only enter the platform if this $u_i^L \geq 0$.

Last lecture we showed that:

$$N^L = \int_{-\infty}^{\infty} \int_{P^L(N^R)-b^L N^R}^{\infty} f^L(b^L, B^L) dB^L db^L$$

and

$$V^L = \int_{-\infty}^{\infty} \int_{P^L-b^L N^R}^{\infty} (b^L N^R + B^L) f^L(b^L, B^L) dB^L db^L$$

where N^L is the number of users in side L and V^L is the welfare of users in side L.

11.2 Overview of this lecture

Assume f is nonzero everywhere. Then increasing $P^L(N^R)$ always decreases the number of users in side L. Furthermore, if we fix some target $\hat{N}^L$ then there is a unique price $P^L(\hat{N}^L, N^R)$ that will result in $\hat{N}^L$ users. We call this $P^L(\hat{N}^L, N^R)$ an **insulating tarriff**.

Typically, $P^L(N^R)$ will look like:

$$P^L(N^R) = P^L + p^L N^R$$

where P^L is a **subscription fee** and p^L is a **transaction fee**.

Today we will look at how to pick P^L to maximize welfare and revenue.

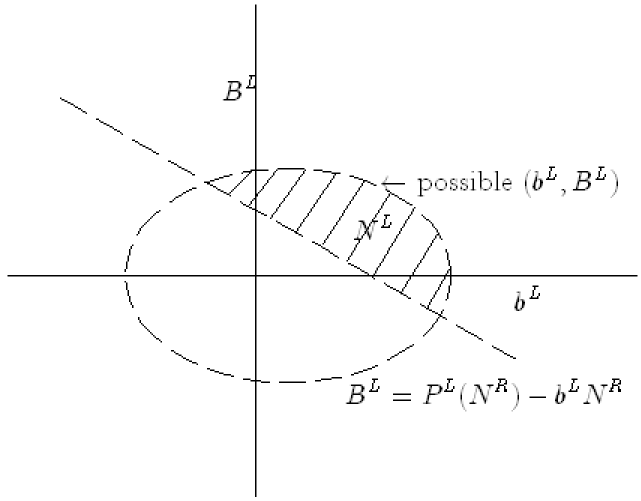

Figure 11.1. Marginal users gain 0 utility from joining the platform. They are users that lie exactly on the dotted line (Figure by Qinru Shi)

11.3 Welfare and Profit Maximization

11.3.1 First order conditions

$$\text{Welfare} = W(N^L, N^R) = V^L + V^R - c^L N^L - c^R N^R - cN^L N^R$$
$$\text{Profit} = \Pi(N^L, N^R) = P^L N^L + P^R N^R - c^L N^L - c^R N^R - cN^L N^R$$

where c^L, c^R, and c are marginal costs incurred to run and maintain the platform.

In a ride-sharing platform c^{riders} may be marketing costs, $c^{drivers}$ the cost of onboarding a new driver, and c the cost of insuring each ride.

The first order condition for welfare maximization is

$$\frac{\partial W}{\partial N^L} = 0 \implies \frac{\partial V^L}{\partial N^L} + \frac{\partial V^R}{\partial N^L} = c^L + cN^R$$

and the first order condition for profit maximization is

$$\frac{\partial \Pi}{\partial N^L} = 0 \implies P^L + N^L \frac{\partial P^L}{\partial N^L} + N^R \frac{\partial P^R}{\partial N^L} = c^L + cN^R$$

11.3.2 Solving for partial derivatives via Leibniz's Rule

1. Recall last lecture we solved for

$$\frac{\partial N^L}{\partial N^L} = 1 \implies \frac{\partial P^L}{\partial N^L} = \frac{-1}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L}$$

We can interpret the integral in the denominator as the mass of people sitting on the margin. The interpretation here is that by perturbing the price of side L up slightly, you lose exactly those people who had 0 utility.

2. We also computed

$$\frac{\partial N^L}{\partial N^R} = 0 \implies \frac{\partial P^L}{\partial N^R} = \frac{\int_{-\infty}^{\infty} b^L f^L(b^L, P^L - b^L N^R) db^L}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L}$$
$$= \tilde{b}^L(N^L, N^R)$$

where $\tilde{b}^L(N^L, N^R)$is the **AIVMU (average interaction value of marginal users)**

3. Applying Lebniz's rule on V^L yields

$$\frac{\partial V^L}{\partial N^L} = -\int_{-\infty}^{\infty} P^L f^L(b^L, P^L - N^R b^L) \frac{\partial P^L}{\partial N^L} db^L = P^L$$

This matches our intuition since the marginal users have utilty 0.

4.

$$\begin{aligned}\frac{\partial V^L}{\partial N^R} &= -\int_{-\infty}^{\infty} P^L f^L(b^L, P^L - N^R b^L)(\frac{\partial P^L}{\partial N^R} - b^L) db^L + \int_{-\infty}^{\infty}\int_{P^L - b^L N^R}^{\infty} b^L f^L(b^L, B^L) dB^L db^L \\ &= 0 + \int_{-\infty}^{\infty}\int_{P^L - b^L N^R}^{\infty} b^L f^L(b^L, B^L) dB^L db^L \\ &= N^L \bar{b}^L\end{aligned}$$

This is the expected interaction benefit b^L over all users in side L and is called the **AIVU (average interaction value of user)**. Intuitively, each user gains $\bar{b}^L$ in interaction benefit whenever someone joins the other side of the platform.

11.3.3 Welfare

Substituting the partial derivatives we computed into the first order condition gives:

$$P_W^L = c^L + cN^R - N^R \bar{b}^R$$

$c^L + cN^R$ is the marginal cost of adding a user to side L. $N^R \bar{b}^R$ can be interpreted as a user's externality, the benefit given to the other side of the platform.

11.3.4 Profit

$$P_{\Pi}^{L} + N^{L}\frac{\partial P^{L}}{\partial N^{L}} = c^{L} + cN^{R} - N^{R}\tilde{b}^{R}$$

Define **price elasticity of demand** as

$$\eta^{L}(N^{L}, N^{R}) = -\frac{\partial N^{L}/N^{L}}{\partial P^{L}/P^{L}}$$

Typically price elasticity is positive since as price increases, customers decrease.
Now we can rewrite

$$P_{\Pi}^{L} = c^{L} + cN^{R} + \frac{P_{\Pi}^{L}}{\eta^{L}} - N^{R}\tilde{b}^{R}$$

$\mu^{L} = \dfrac{P_{\Pi}^{L}}{\eta^{L}}$ is called the **Cournot distribution** or **market power**.
We can also look at the difference between these two prices:

$$P_{\Pi}^{L} - P_{W}^{L} = \mu^{L} + N^{R}(\tilde{b}^{R} - \bar{b}^{R})$$

11.3.5 Examples

Rouchet-Tirole (2003)

Assume $B^{L}, B^{R}, c^{L}, c^{R} = 0$, i.e. no membership benefits or costs. Then

$$N^{L}(N^{R}, P^{L}) = 1 - F^{L}(\frac{P^{L}}{N^{R}})$$

$$P^{L}(N^{R}, \hat{N}^{L}) = (F^{L})^{-1}(1 - \hat{N}^{L})N^{R}$$

The only fee is a per transaction fee $p^{L} = (F^{L})^{-1}(1 - \hat{N}^{L})N^{R}$.

Armstrong (2000)

Assume $c = 0$ and b^{L}, b^{R} fixed. Then

$$N^{L} = 1 - F^{L}(P^{L} - b^{L}N^{R})$$

$$P^{L}(N^{R}, \hat{N}^{L}) = (F^{L})^{-1}(1 - \hat{N}^{L}) + b^{L}N^{R}$$

Scaled income model

$u_{i}^{L} = b_{i}^{L}(N^{R} + \dfrac{1}{\beta^{L}})$ where $\beta^{L} = \dfrac{B^{L}}{b^{L}}$