The pricing problem

- Single good, single buyer with value $V \sim F$
- Can set price p, buyer buys if $V \geq p$
- $$\boxed{R^* = \max_{p \geq 0} p \cdot (1 - F(p))}$$

$$= \max_{q \in [0,1]} F^{-1}(1-q) \cdot q$$

Myerson price p^* : $\boxed{p^* - \dfrac{1-F(p^*)}{f(p^*)} = 0}$

(F is regular iff $p - \left(\dfrac{1-F(p)}{f(p)}\right)$ non-decreasing in p)

- To go beyond this, we need to define 3 things:
  1) Behavioral model of buyer
  2) Objectives and constraints of seller
  3) Structure of available information

- The rest of the course considered several such settings

- Buyer model - perfect segmentation - probabilistic choice
  - strategic behavior
- Seller models - limited capacity
  - admission control (fare-classes)
  - network externalities
  - (dynamic pricing)
  - DSIC mechanisms
  - 2-sided marketplace platforms
- Information structure - Full knowledge of buyer value/choice distributions
  - Learning from data (spiral-down!)
  - No knowledge (DSIC mechanisms, Bulow-Klemperer)

Main ideas and techniques

1) DP formulation for pricing problems (value function, Bellman equation, optimal control)

2) Protection level policies (for single-resource allocation)

(2.5). Convexity, Jensen's inequality)

3) Fluid approximations for complex DPs (and the bid-price heuristic)

4) The spiral-down effect (importance of using the correct model, effect of improper learning)

5) Probabilistic choice models for buyer behavior : Luce's axioms and the MNL

6) Assortment optimization under the MNL model - optimality of nested-by-revenue sets

7) Mechanism design - The Vickrey auction, dominant strategy incentive compatibility (DSIC).

8) Myerson's Lemma - DSIC $\Leftrightarrow$ monotone allocation rule (for single parameter settings)

9) Optimal revenue DSIC mechanism $\Rightarrow$ maximize 'virtual welfare' (reserve prices, Bulow-Klemperer theorem)

10) 2-sided marketplace optimization - choose insulating prices $P^L(N^L, N^R)$, $P^R(N^L, N^R)$; optimize over N^L, N^R

Beyond single parameter settings - things get strange!

Eg - Single buyer, 2 non-identical items

- Values $v_1, v_2 \sim F$ iid, additive utilities

1) $v_1, v_2 \sim \begin{cases} 1 & \text{wp } 0.5 \\ 2 & \text{wp } 0.5 \end{cases}$

- Sell both separately $\Rightarrow R = 2$ (for $p = 1$ or 2)
- Sell bundle at price 3 $\Rightarrow R = 3 \cdot (1 - 1/4)$ $= 9/4$

buyer buys if v_1, v_2 both not equal to 1

2) $v_1, v_2 \sim \begin{cases} 0 & \text{wp } 1/3 \\ 1 & \text{wp } 1/3 \\ 2 & \text{wp } 1/3 \end{cases}$

- Sell separately (at $p = 1$ or 2) $\Rightarrow R = 4/3$
- Sell bundle (at $p = 2$) $\Rightarrow R = 4/3$

$v_1 \backslash v_2$	0	1	2
0	0	0	2
1	0	0	3
2	2	3	3

- Offer 1 item at $p = 2$ or bundle at $p = 3$

$R = 13/9 > 4/3$