ORIE 4154: Pricing and Market Design **Spring 2017**

Lecture 23

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23.1 Overview

Until now, we have looked at *monopolist pricing problems*: we assume that a firm wants to sell some items, and is setting prices for these so as to maximize revenue. In this section, we will instead consider the price theory of 2-sided platforms – we will look at how to model a marketplace platform, which facilitates some form of trade between two distinct groups of agents. The main idea is that in such platforms, the utility of any agent often depends on the number of agents on the other side. Moreover, the platform can set prices for both the sides, but needs to account for these cross-market *externalities* in order to maximize revenue.

Examples of 2-sided platforms

First, to get a feel for what we will talk about, here are some real-life examples of 2-sided platforms:

1. Newspaper: Reader v.s. Advertiser. Newspapers get paid by advertisers by putting up advertisements, but may lose readers if they have too many advertisements.

2. Credit Card Companies: Customers v.s. Merchants. Customers generally prefer credit cards and are likely to purchase more when using credit cards, but credit card companies charge a per transaction fee from merchants and merchants might increase the price for credit card customers or set lower bounds for credit card purchases.

3. Ridesharing companies: Drivers v.s. Riders. Riders want low prices and low wait-times; the latter necessitates more drivers. On the other hand, drivers care about how much they make on average, and so more riders means less idle-time.

4. (Heterosexual) Dating and matchmaking sites: Men v.s. women. The utility of the platform for agents of any sex depend on the number of opposite sex agents on the platform.

23.2 Price theory of 2-sided platforms

One useful approach for thinking about 2-sided platforms was initiated by Rochet and Tirole [1]; we will follow a generalized version of these ideas, taken from a paper by Weyl [2].

The basic model of a 2-sided platforms

To model the marketplace, we want to capture the fact that the outcomes of a transaction in a 2-sided marketplace depend not only on value or cost but also on the **payment structure**; for example, a platform may have an entry/membership fee, and/or a transaction fee, etc. In particular, The model proposed by Weyl in [2] is structured as follows:

- Two Sides: L (left) and R (right).
- Number of people N^L, N^R on sides L, R are normalized to lie in $[0,1]$. In other words, think of these quantities as probabilities that agents on either side join the marketplace. In particular, suppose there are n^L, n^R potential agents on sides $L, R-$ then the number of people who join on each side is $N^L \cdot n^L$ and $N^R \cdot n^R$.
- Users of side L (similarly R) derive 2 benefits from the platform. (Both values are scaled and can be negative.)

  (a) Membership benefit B^L (similarly B^R),

  (b) Interaction benefit b^L (similarly b^R)

  Agents on side L have benefit parameters (B^L, b^L) drawn randomly from some distribution F^L (similarly for side R, with F^R)

In this model, the utility functions for a random agent on each side of the market are:

$$U^L = B^L + b^L N^R - P^L(N^L, N^R)$$
$$U^R = B^R + b^R N^L - P^R(N^L, N^R)$$

An agent joins the platform iff her utility is positive; note that this however depends on the number of agents on the other side of the market. In particular, we have the following (these correspond to IR constraints for agents on each side):

$$N^L = \mathbb{P}\left[B^L + b^L N^R \geq P^L(N^L, N^R)\right]$$
$$N^R = \mathbb{P}\left[B^R + b^R N^L \geq P^R(N^L, N^R)\right]$$

Note that all the benefits can be positive or negative (i.e., benefits or cost); similarly, prices can be positive or negative, the latter corresponding to making a payment to an agent. Note also that we are assuming that there are only homogeneous cross-network interactions, and that there is no price discrimination and/or externalities within a side.

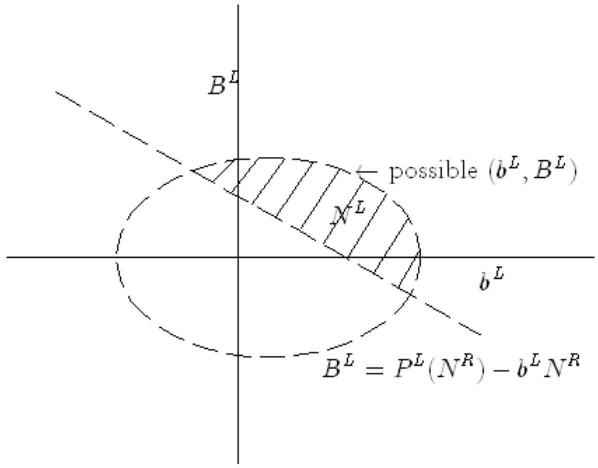

Figure 23.1. Loyal and Marginal Users

Analysis of the model

Suppose the distribution of the benefits $F^L(B^L, b^L)$ and $F^R(B^R, b^R)$ have density functions f^L and f^R, which lie (i.e., are non-zero) on some convex set in $\mathbb{R}^2$. We can re-write the IR constraints as

$$N^L = \int_{-\infty}^{\infty} \int_{P^L(N^L,N^R)-b^L N^R}^{\infty} f^L(b^L, B^L) dB^L db^L$$
$$N^R = \int_{-\infty}^{\infty} \int_{P^R(N^L,N^R)-b^R N^L}^{\infty} f^R(b^R, B^R) dB^R db^R \tag{23.1}$$

As shown in Figure 23.1, the probability of the shaded area represents N^L. We call users on the dotted line 'marginal users' since they have 0 utility; users in the shaded are referred to as 'loyal users'.
The system can have multiple equilibriums (i.e. values of (N^L, N^R) which jointly satisfy Eqn 23.1). For example, let $b^L = b^R = 1$, $B^L = B^R = 0$ and $P^L = P^R = \frac{1}{2}$. Then the system reaches equilibrium when $(N^L, N^R) = (0, 0)$ or $(1, 1)$. Our aim is to choose prices (and equilibrium sizes N^L, N^R) such that under the resultant equilibrium, we maximize profits.

Insulating Prices

In order to solve the problem, the main idea is to set (N^L, N^R) as free variables, and then set *insulating prices* $P^L(N^L, N^R)$ and $P^R(N^L, N^R)$ so as to achieve these desired levels.
To see how to do this graphically, observe that whenever we increase P^L (while keeping N^R fixed), the dotted line in Figure 23.1 shifts up and N^L decreases. Formally, this shows that $\frac{\partial}{\partial P^L} N^L(N^R, P^L) < 0$; as a result, we have that the insulating price $\tilde{P}^L(N^L, N^R)$ is well defined as the inverse of $N^L(N^R, P^L)$. Similarly, we have $\tilde{P}^R(N^L \mid N^R)$.

Examples

Rouchet-Tirole (2003) Assume $B^L, B^R, c^L, c^R = 0$, i.e. no membership benefits or costs; $b^L \sim F^L$ and $b^R \sim F^R$. Then

$$N^L(N^R, P^L) = 1 - F^L\left(\frac{P^L}{N^R}\right)$$
$$\Rightarrow \tilde{P}^L(N^L, N^R) = (F^L)^{-1}(1 - N^L) \cdot N^R$$

The insulating price is this a *per transaction fee* $p^L = (F^L)^{-1}(1 - \hat{N}^L)$.

Armstrong (2000) Assume $c = 0$ and b^L, b^R fixed. Then

$$N^L(N^R, P^L) = 1 - F^L\left(\frac{P^L - b^L N^R}{N^R}\right)$$
$$\Rightarrow \tilde{P}^L(N^L, N^R) = (F^L)^{-1}(1 - N^L) + b^L N^R$$

This now comprises of a membership fee $(F^L)^{-1}(1 - N^L)$ and a per-transaction fee of b^L.

Profit Maximization

Now, we want to choose (N^L, N^R) and maximize various objectives under the resulting insulating prices. In particular, we can write the profit as

$$\text{Profit} = \Pi(N^L, N^R) = P^L N^L + P^R N^R - c^L N^L - c^R N^R - cN^L N^R$$

where c^L, c^R, and c are marginal costs incurred to run and maintain the platform. For example, in a ride-sharing platform c^{riders} may be marketing costs, $c^{drivers}$ the cost of onboarding a new driver, and c the cost of insuring each ride.
The first order condition for profit maximization is

$$\frac{\partial \Pi}{\partial N^L} = 0 \implies P^L + N^L \frac{\partial P^L}{\partial N^L} + N^R \frac{\partial P^R}{\partial N^L} = c^L + cN^R$$

To find the optimal prices, we need to simplify the partial derivatives.

Solving for partial derivatives via Leibniz's Rule

1. Using Leibniz integral rule, we can differentiate (23.1) with respect to N^L, to get

$$\frac{\partial N^L}{\partial N^L} = 1 = \left(-\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L\right) \cdot \left(\frac{\partial P_L}{\partial N^L}\right)$$
$$\Rightarrow \frac{\partial P^L}{\partial N^L} = -\frac{1}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R) db^L}$$

   We can interpret the integral in the denominator as the mass of people sitting on the margin. The interpretation here is that by perturbing the price of side L up slightly, you lose exactly those people who had 0 utility.

2. Next, since we are fixing N^L, N^R, we can differentiate (23.1) with respect to N^R to get

$$\frac{\partial N_L}{\partial N^R} = \int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R)(\frac{\partial P_L}{\partial N^R} - b^L)db^L$$
$$\Rightarrow \frac{\partial P^L}{\partial N^R} = \frac{\int_{-\infty}^{\infty} b^L f^L(b^L, P^L - b^L N^R)db^L}{\int_{-\infty}^{\infty} f^L(b^L, P^L - b^L N^R)db^L}$$

Let $\tilde{b}^R(N^L, N^R)$ represent this *average interaction value of marginal users*, i.e., $\frac{\partial P^L}{\partial N^R} = \tilde{b}^R(N^L, N^R)$. We can use this to write

$$P^L(N^L, N^R) = c^L + cN^R - N^R \cdot \tilde{b}(N^L, N^R)$$

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Profit maximization

$$P_{\Pi}^L + N^L \frac{\partial P^L}{\partial N^L} = c^L + cN^R - N^R \tilde{b}^R$$

Define **price elasticity of demand** as

$$\eta^L(N^L, N^R) = -\frac{\partial N^L/N^L}{\partial P^L/P^L}$$

Typically price elasticity is positive since as price increases, customers decrease.
Now we can rewrite

$$P_{\Pi}^L = c^L + cN^R + \frac{P_{\Pi}^L}{\eta^L} - N^R \tilde{b}^R$$

$\mu^L = \dfrac{P_{\Pi}^L}{\eta^L}$ is called the **market power**. We note that we derive a similar expression for choosing a single item to maximize profits (HW 1), except that the optimal price now has an added contribution of the form $-N^R \tilde{b}^R$, that captures the network externality.

Bibliography

[1] Rochet, J.C., Tirole, J. *Platform Competition in Two-Sided Markets*, IDEI Working Papers 152, Institut d'conomie Industrielle (IDEI), Toulouse, 2003.

[2] Weyl, E Glen. *A Price Theory of Multi-sided Platforms*, American Economic Review, 100(4): 1642-72, 2010.