Models of customer behavior

- Suppose we want to sell m different items among n buyers
  - how do buyers choose items?
- 3 models of customer behavior

i) Perfect segmentation - Each customer only wants a single item from the set of items

ii) (Probabilistic) Choice model - Each customer chooses an item from amongst the displayed items

iii) Strategic choice - Customers compete with each other to try and get the 'best deal' for themselves.

Auctions and mechanism design

- Up till now, we looked at perfect segmentation and probabilistic choice, and used pricing, capacity control and assortment control as our optimization tools.
- We now introduce a model for strategic customers, and a new optimization tool - auctions
- Consider a setting where we want to sell 1 item
- Quasilinear utility model
  - Each bidder i has an independent value v_i for the item. This value is private
  - If the bidder is offered the item at price $p \leq v_i$, then its utility is $v_i - p$

    Sale

    If the bidder is not offered the item (or offered at price $p > v_i$), then its utility is 0

    No sale

- Sealed-bid auctions

These occur in three steps.

i) (Bidding) Each bidder i communicates bid b_i to seller

ii) (Allocation Rule) Seller chooses bidder who gets the item (if anyone)

iii) (Payment Rule) Seller decides on price

— Natural allocation rule - sell to highest bidder

— Payment rule ? This affects bidder behavior!

Eg - What if price $= 0$?

Then everyone tries to set b_i as high as possible!

* First-price auctions

- Set payment $p = \max_i [b_i]$

  Allocate item to $i^* = \arg\max_i [b_i]$

- Problem: Very difficult for bidders to decide their bid!

* Second-price auction (Vickrey auction)

- Allocate item to $i^* = \arg\max_i [b_i]$

  Set payment $p = \max_{j \neq i^*} [b_j]$

- This is equivalent to an ascending price auction

- Now what should bidder i bid?

- We now show two properties of the Vickrey auction

i) In the Vickrey auction, every bidder i sets her bid b_i = private valuation v_i, no matter what the other bidders do (dominant strategy)

Pf: Let $b_{-i} \equiv$ vector of bids of all bidders other than i

- Fix some arbitrary bidder i, valuation v_i, bids b_{-i}
- Let $B = \max_{j \neq i} b_j$, and suppose i knows B
- There are 2 cases
    i) If $v_i < B$, then bidder i can get a utility of $\max\{\underbrace{0}_{\text{bid } b_i < B}, \underbrace{v_i - B}_{\text{bid } b_i > B}\} = 0$, which can be achieved by setting $b_i = v_i$
    ii) If $v_i \geq B$, then bidder i can get utility of $\max\{0, v_i - B\} = v_i - B$, again by setting $b_i = v_i$

2) In the Vickrey auction, every truthtelling bidder has non-negative utility

Pf - If bidder i loses, then utility $= 0$

- If bidder i wins (while bidding $b_i = v_i$), then utility is $v_i - B \geq v_i - b_i = 0$ (as $b_i \geq B$)

Henceforth, we want all mechanisms to have these 2 properties

- $\boxed{\text{(Dominant Strategy) Incentive Compatibility (DSIC)}}$

  Bidder i's utility is maximized by setting $b_i = v_i$, no matter what other bidders bid.

- $\boxed{\text{Individual Rationality (IR)}}$

  Every bidder has non-negative utility assuming truthtelling.

- As shorthand, we will call a mechanism with these two properties to be DSIC. Our aim is to design DSIC mechanisms to maximize some given objective.